HSE/Math in Moscow 2015-2016// Topology 2 // Problem sheet 7

The cohomology of projective spaces

Question 1. Calculate the groups $H^*(\mathbb{C}P^n, G)$ for $G = \mathbb{Z}, \mathbb{Z}/n, \mathbb{Q}$ using (a) cellular chain complexes; (b) the Universal coefficient formula, and compare the results.

Question 2. Same as Question 1, but for $\mathbb{R}P^n$ instead of $\mathbb{C}P^n$.

Question 3. Same as Question 4 from HW 6 but for cohomology, instead of homology, with $\mathbb{Z}$ coefficients.

Borsuk-Ulam continued

Question 4. Recall that the Borsuk-Ulam theorem has the following corollary: for every continuous self-map $f : S^n \to \mathbb{R}^n$ there is an $x \in S^n$ such that $f(-x) = f(x)$. Deduce that if S^n is represented as the union of $n+1$ closed sets, then one of these will contain a pair of opposite points of S^n. [Hint: distance functions.]

Question 5. Let $X_1, \ldots, X_n$ be compact measurable subsets. Let us also assume they are sufficiently nice, meaning that if $f : \mathbb{R}^n \to \mathbb{R}$ is an affine function then the measure of $X_i \cap f^{-1}(0, \infty)$ depends continuously of f. You may assume e.g. that every X_i is a compact polyhedron.

Prove that there is a hyperplane $\subset \mathbb{R}^n$ which divides every X_i into two parts of equal measure.

Problems for discussion

None this week.